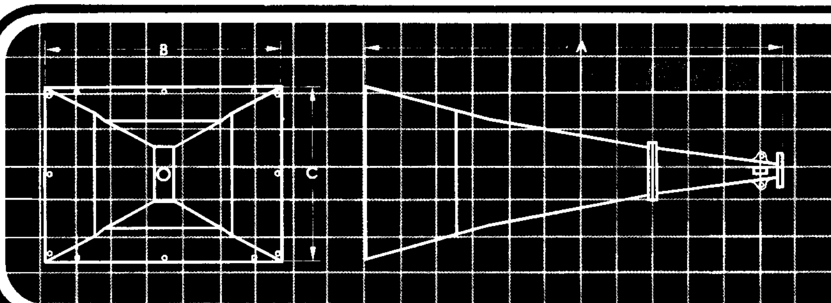

ALTEC LANSING ENGINEERING NOTES

Tech. Letter No. 264A

Using Thiele-Small Parameters

by Rex Sinclair

In order to properly design an enclosure for a loudspeaker, certain parameters of the loudspeaker should be known. The important parameters for enclosure design are:

1) the free air resonant frequency (f_S),

2) the total Q (Q_T),

3) the equivalent volume (V_{AS}), (not the actual physical volume).

Calculator program CP-22A is available as an alternative or as a supplement to some of the design methods given here.

Maximally flat alignments

The combination of enclosure volume and tuning (the alignment) chosen for a given loudspeaker is frequently of the kind first given by Thiele [1] which are usually called maximally flat alignments. The enclosure volume (V_B) for alignments of this kind can be found from the graph shown in Figure 1. The physical volume of the loudspeaker should be added to this.

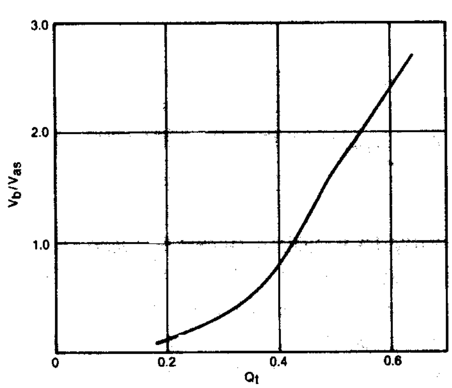

Figure 1. The dependence of V_b on Q_t.

The frequency (f_B) to which the enclosure should be tuned is given in Figure 2 and the frequency (f_3) for 3 dB below the assymptotic level is given in Figure 3. Once V_B and f_B are known, the port area A_p in a 0.75 in. baffle can be found from Figure 4.

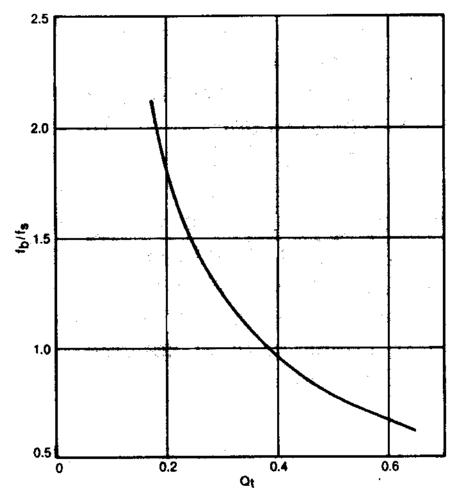

Figure 2. The dependence of f_b on Q_t for V_b given in Figure 1.

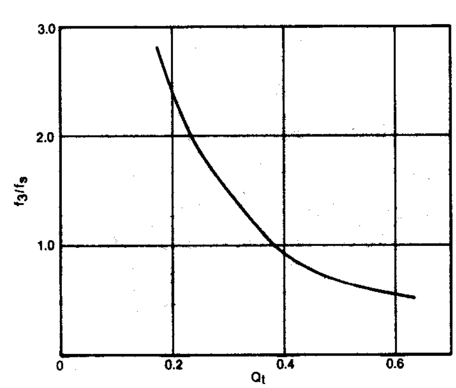

Figure 3. The dependence of f_3 on Q_t for enclosures designed according to Figures 1 and 2.

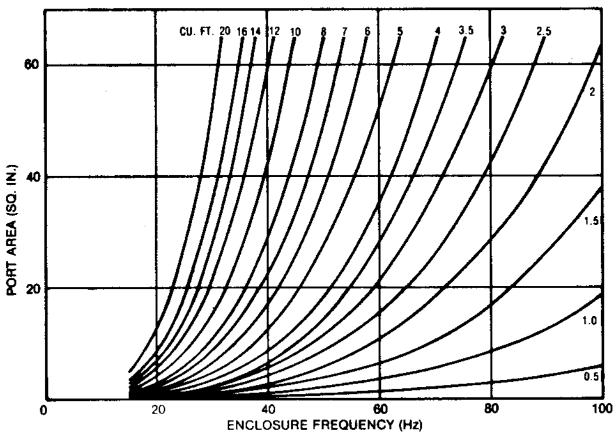

Figure 4. The dependence of A_p on V_b and f_b.

If the required port is too large to be on the graph or to fit in the baffle without undermining the strength, multiple ports having a smaller total area can be used. Supposing that in a given instance it is decided to use n ports, then the area of each can be found by using Figure 4 with V_B/n for the volume. If on the other hand it is decided that the area is too small and will cause whistling or distortion, it can be converted into a larger area duct. The volume lost to the duct is given in Figure 5 and should be added to V_B (along with the volume occupied by the loudspeaker). The duct length can be found from Figure 6. By the combined use of Figures 4, 5 and 6, any configuration of equal sized ports can be converted to any other number of equal ducts and vice versa. If a duct has excessive length for the cabinet depth, a smaller duct area and/or a smaller number of ducts should be used. Examples are given later. Typical loudspeaker physical volumes and Thiele-Small parameters for a selection of Altec Lansing loudspeakers are given in Tech Letter 267. The equations used to create Figures 1, 2, 3, 4 and 6 are given in the Appendix.

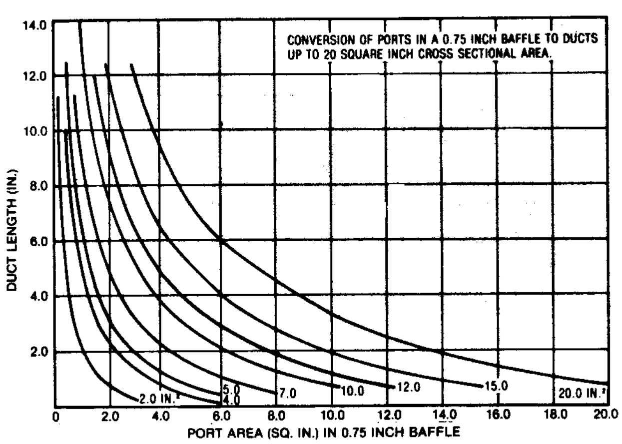

Figure 5. The dependence of duct volume on duct area and equivalent port area (A_p) in a 0.75 in. baffle.

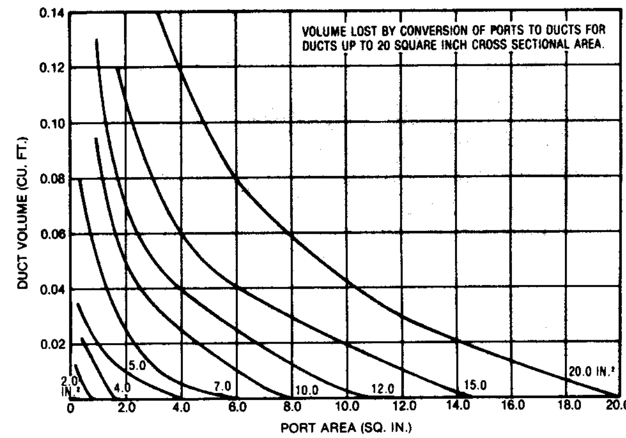

Figure 6. The dependence of duct length on duct area and A_p.

Non-Thiele alignments

Frequently it is desired to use a loudspeaker in an existing cabinet and to port it for the best performance available for that size. In which case the port area must be found. It is also desirable to know the − 3 dB frequency and the smoothness of response. In order to characterize the smoothness, the height of the bump in the response curve has been calculated. Some values of the bump in dB are given as negative. In cases having a negative bump, an auxiliary positive bump is always present at a lower frequency. The difference between positive and negative bumps is shown in Figure 7.

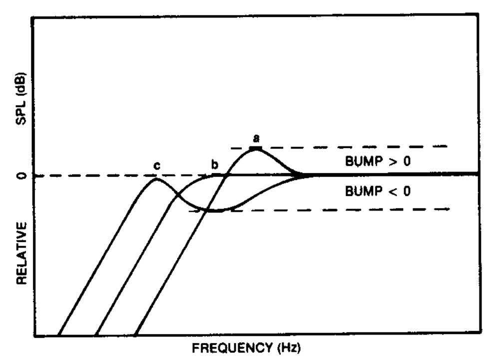

Figure 7. Diagramatic illustration of response curves for volumes a) smaller than maximally flat, b) maximally flat and c) larger than maximally flat.

The frequency to which a given enclosure should be tuned for a loudspeaker of known parameters can be found from Figure 8 where a family of lines is given for different values of Q_T. Tunings for intermediate values can be found by interpolation. The required port area can then be found from Figure 4 after subtracting the loudspeaker physical volume.

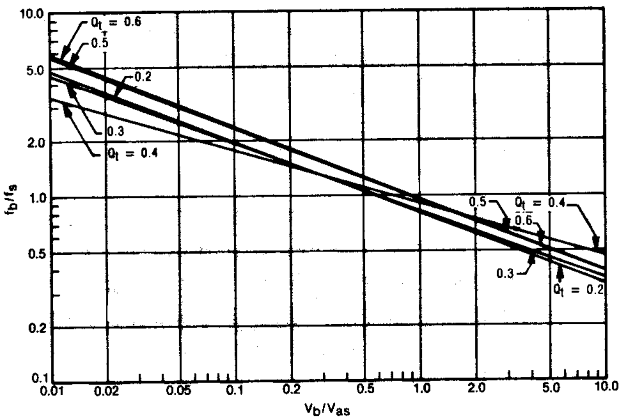

Figure 8. The Dependence of Enclosure Tuning Frequency on Volume for Different Values of Q_T.

For conversion to multiple ports and ducts, the procedure is the same as that given above for maximally flat alignments. Where it is considered desirable to convert ports into ducts, the duct area is selected and its volume found from Figure 5. This volume is then subtracted from the enclosure volume and a new port area found. This can be repeated until there is no significant change in the effective enclosure volume. The duct length can now be found from Figure 6.

Expected performance of such a system can also be predicted. The value of f_3 can be found from Figure 9 with interpolation where necessary. The size of any bump present can be found from Figure 10. For the higher Q_T values with larger volumes, the auxiliary positive bump may exceed the assymptotic SPL (0 dB) in which case its value can also be found from Figure 10. An estimate of frequency at which the main bump occurs can be found from Figure 11 and the frequency (f_{AB}) of the auxiliary positive bump when greater than 0 dB can be found from Figure 12. Equations for Figures 8 and 9 are given in the Appendix.

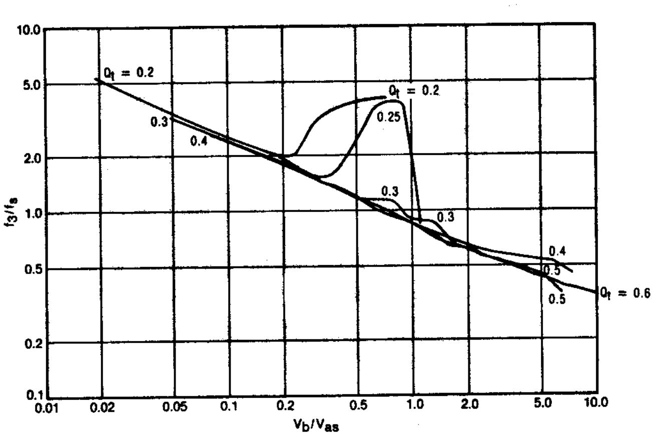

Figure 9. The Dependence of f_3 on Enclosure Volume Tuned According to Figure 8.

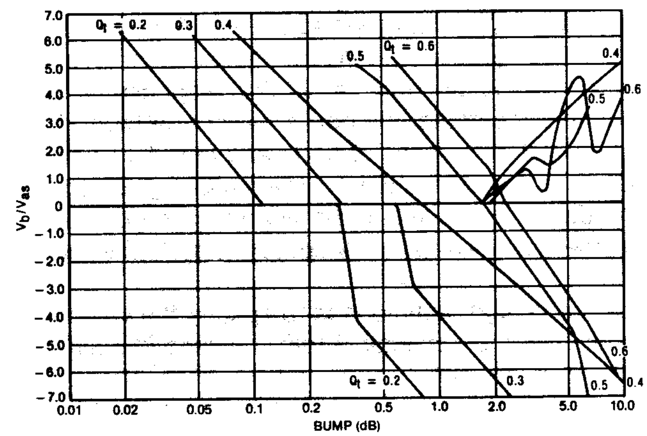

Figure 10. The Dependence of Response Curve Bump Magnitude on Enclosure Volume Tuned According to Figure 8.

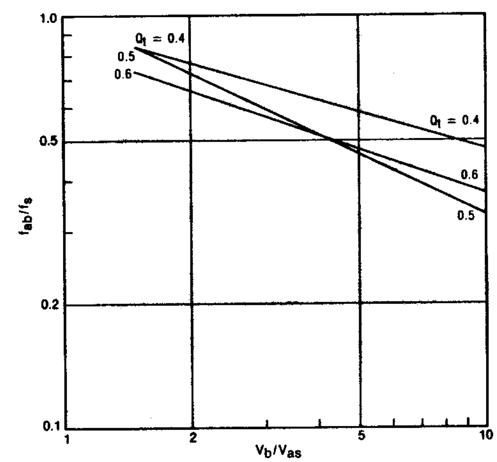

Figure 11. The Dependence of Bump Frequency on Enclosure Volume Tuned According to Figure 8.

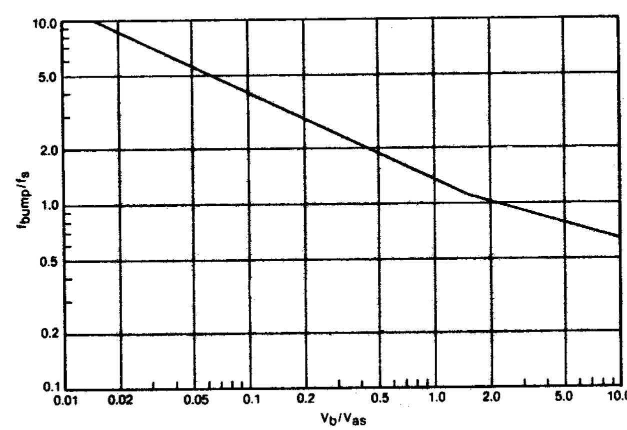

Figure 12. The Dependence of the Frequency of the Auxiliary Positive Bump, when Present and Greater than Zero, on Enclosure Volume Tuned According to Figure 8.

Another design technique is to specify the −3 dB frequency required and find the required enclosure volume from Figure 9. The response curve bumps and their frequencies can be found from Figures 10, 11 and 12. A decision can then be made whether or not the design is acceptable. If the response is considered acceptable, the port area can be found from Figure 4. The number of ports and any duct conversion can be accomplished in the manner given above. Any duct volume should be added to the enclosure volume along with the loudspeaker physical volume.

Another design technique is to specify the value of a negative bump acceptable for an extended bass or a positive bump required for a false bass effect. The enclosure volume can be found from Figure 10. The rest of the design follows the methods outlined above.

Yet another design technique is to use a new set of extended bass (X BASS) alignments. These alignments give a slower low frequency initial roll-off than Thiele alignments and hence lower values of f_3 and f_6 usually with less acoustical masking at f_3 and f_6. The design parameters for these alignments are given by

$$V_B/V_{AS} = 7.95\, Q_T^{2.21} \qquad (1)$$

and

$$f_B/f_S = 0.471\, Q_T^{-0.677} \qquad (2)$$

The predicted value of f_3 is given by,

$$f_3/f_S = 0.21\, Q_T^{-1.46} \text{ for } Q_T \leqslant 0.366 \qquad (3)$$

and

$$f_3/f_S = 0.33\, Q_T^{-1.01} \text{ for } Q_T > 0.366 \qquad (4)$$

These relationships are shown graphically in Figure 13.

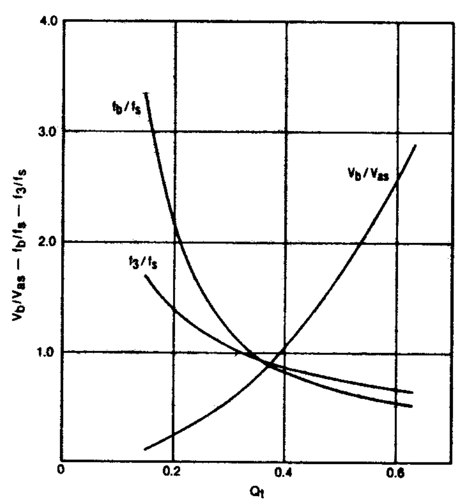

Figure 13. The Dependence of Enclosure Volume and Tuning Frequency on Q_t and the Predicted f_3 for a System of Extended Bass (Non-Thiele) Alignments.

For Q_T values up to 0.4, these alignments give no ripple. For higher Q_T values there is a slight ripple similar to that of the Thiele alignments at higher Q_T values. As Q_T is increased, these alignments become closer to being Thiele alignments.

Sixth order alignments

Another completely different family of alignments is available wherein the frequency response of the system is equalized by an auxiliary filter which is normally in the amplifier input circuit to yield a smooth extended bass. The possibilities are infinite. Discussion here is restricted to filters giving a single pass band of additional boost, namely sixth order systems. In order to provide protection from cone excusion damage, discussion is further limited to peak boost frequencies in the vicinity of f_B where cone excusion is low and to boosts of 6 dB. In the interests of simplicity the auxiliary filter is restricted to a quality factor (Q_A) value of 2.

An approximation to a maximally flat sixth order alignment can be obtained by using an enclosure volume (V_{B6}) given by Keele [2] i.e.,

$$V_{B6} = 4.1\, V_{AS}\, Q_T^2 \qquad (5)$$

tuned to a frequency given by

$$f_{B6} = 0.3\, f_S/Q_T. \qquad (6)$$

The port area can be found from Figure 4. Any port and duct manipulations are the same as before. The auxiliary filter has 6 dB boost centered at f_P and a Q_A of 2. The peaking frequency frequency (f_P) is given by

$$f_P = 1.07\, f_{B6} \qquad (7)$$

and the -3 dB frequency is given by

$$f_3 = f_P. \qquad (8)$$

These relationships are shown graphically in Figure 14.

Another sixth order approximate design method [2] yielding systems which are not maximally flat is to take any existing alignment (e.g. Thiele or X BASS) and to retune the enclosure to the frequency given by (6) or Figure 14.

The amplifier peaking frequency (f_P) is given by (7) and f_3 is given approximately this time by (8). Alternatively, f_P and the approximate value of f_3 can be found from Figure 14. Using this technique it is possible to design dual purpose enclosures, i.e. a Thiele or X BASS alignment with an alternative duct for sixth order applications. The reverse design philosophy can also be applied.

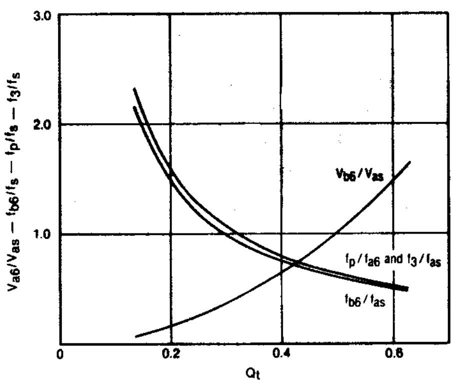

Figure 14. The Dependence of Enclosure Volume, Tuning Frequency, Auxiliary Filter Peaking Frequency and f_3 on Q_t for Maximally Flat Sixth Order Systems.

For maximally flat systems designed according to (5) and (6), the sixth order enclosure is smaller than a Thiele enclosure for Q_T values greater than 0.338 and larger for Q_T values less than 0.338.

Examples

1) To design a maximally flat enclosure for a 15 in. loudspeaker having f_S = 25.3 Hz, Q_T = 0.21, V_{AS} = 17.9 cu. ft. and physical volume = 0.17 cu. ft.

From Figure 1,

$$V_B/V_{AS} = 0.137, \qquad (9)$$

therefore $V_B = 2.45\ \text{ft.}^3$ (10)

and from Figure 2,

$$f_B/f_S = 1.73 \qquad (11)$$

therefore $f_B = 43.72$ Hz. (12)

The −3 dB frequency can be found from Figure 3 which yields

$$f_3/f_S = 2.26, \qquad (13)$$

therefore $f_3 = 57.08$ Hz. (14)

From Figure 4, for 43.72 Hz and 2.45 cu. ft., the port area

$$A_P = 5.20\ \text{in.}^2 \qquad (15)$$

in a 0.75 in. thick baffle. It is now decided that this port is too small for a 15 in. loudspeaker and that a duct should be used. Choosing a duct of 12.568 sq. in. cross sectional area, Figure 5 gives a duct volume of 0.03 cu. ft. The total volume (V_T) is now given by

$$V_T = 2.45 + 0.17 + 0.03\ \text{ft.}^3 \qquad (16)$$

$$= 2.65\ \text{ft.}^3 \qquad (17)$$

From Figure 6, the duct length is 3.75 in.

2) To tune a 5.0 cu. ft. enclosure for use with a 15 in. loudspeaker using two ducts of 7 in.^{2} each. The loudspeaker parameters are f_S = 24.1 Hz, Q_T = 0.24, V_{AS} = 17.0 and the physical volume is 0.17 cu. ft.

Available box volume $V_B = 5.0 - 0.17$

$$= 4.83\ \text{cu. ft.} \qquad (18)$$

therefore $V_B/V_{AS} = 0.284$ (19)

From Figure 8,

$$f_B/f_S = 1.40 \qquad (20)$$

therefore $f_B = 1.40 \times 24.1$ (21)

$$= 33.74\ \text{Hz.}$$

From Figure 4,

$$A_P = 7.0\ \text{in.}^2 \qquad (22)$$

For two ports, i.e. 2.50 ft.^{2} per port, Figure 4 gives a new port area

$$A_{P2} = 2.00\ \text{in.}^2 \qquad (23)$$

From Figure 5, the volume of each duct will be 0.025 cu. ft. A new value of V_B is given by

$$V_B = 5.00 - 0.050 - 0.17\ \text{ft.}^3 \qquad (24)$$

$$= 4.78\ \text{ft.}^3 \qquad (25)$$

As this is not significantly different from the original value of 4.83 cu. ft. the duct length can now be found from Figure 6 and is in fact 3.1 in. The −3 dB frequency and the bump are given by Figures 9 and 10.

By interpolation

$$f_3 = 43.4\ \text{Hz.} \qquad (26)$$

and Bump = 0 dB. (27)

From Figure 11, if there were a bump it would occur at

$$f_B = 58.6 \text{ Hz.}$$

3) To design an X BASS enclosure for the loudspeaker of Example 1 using a 12.568 in.^{2} duct.

From Figure 13,

$$V_B/V_{AS} = 0.27 \quad (28)$$

$$V_B = 4.83 \text{ ft.}^3 \quad (29)$$

$$f_B = 1.35 \times 25.3 \quad (30)$$

$$= 34.16$$

and $$f_3 = 51.36 \text{ Hz.} \quad (31)$$

From Figure 4,

$$A_P = 7.0 \text{ in.}^2 \quad (32)$$

The bump is of course 0 dB.

From Figure 5, the duct volume is 0.021 cu. ft.

The total enclosure volume is now 4.83 + 0.021 + 0.17 cu. ft. i.e.

$$V_T = 5.02 \text{ ft.}^3 \quad (33)$$

From Figure 6, the duct length is 2.6 in.

A frequency response curve is shown for this alignment in Figure 15 along with the standard Thiele alignment of Example 1.

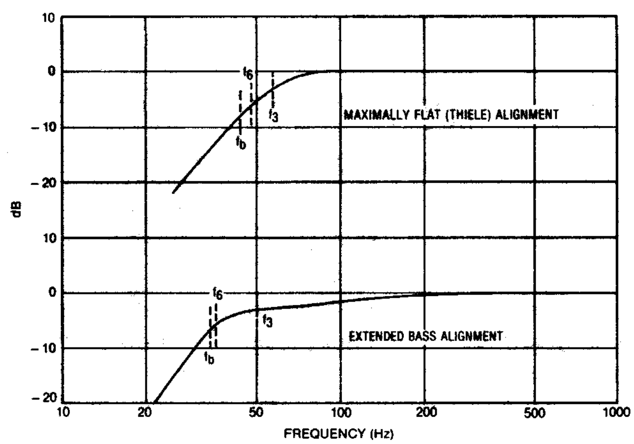

Figure 15. A comparison of Frequency Response of a Maximally Flat Alignment and an X BASS System using the Same Loudspeaker.

4) To design a maximally flat sixth order system using the loudspeaker of Example 1.

From Figure 14,

$$V_{B6}/V_{AS} = 0.18 \quad (34)$$

therefore $$V_{B6} = 3.22 \text{ cu. ft.} \quad (35)$$

$$f_{B6} = 36.05 \text{ Hz.} \quad (36)$$

$$f_3 = f_P = 38.58 \text{ Hz} \quad (37)$$

From Figure 4,

$$A_P = 4.5 \text{ in.}^2 \quad (38)$$

To convert the small port into a duct of 12.568 in.^{2} area, Figure 5 gives a duct volume of 0.034 cu. ft. From Figure 6, the duct length is 4.3 in. This system of course has zero bump when used with an amplifier giving 6 dB boost at 38.58 Hz with a Q_A of 2.

$$V_T = 3.22 + 0.034 + 0.17 \quad (39)$$

$$= 3.42 \text{ cu. ft.} \quad (40)$$

5) To design an alternative turning for the enclosure of Example 3 to give a sixth order system using the same loudspeaker.

$$V_B = 4.83 \text{ cu. ft.} \quad (41)$$

From (6) or Figure 14,

$$f_{B6} = 36.05 \text{ Hz.} \quad (42)$$

and from (7) and (8) or Figure 14

$$f_P = 38.58 \text{ Hz.} \quad (43)$$

and $$f_3 \simeq 38.58 \text{ Hz.} \quad (44)$$

From Figure 4, a frequency of 36.05 Hz requires a port of 4.5 in.^{2}. For a duct of 12.568 in.^{2}, Figure 5 gives a duct volume of 0.034 cu. ft. The duct length from Figure 6 is 4.3 in.

As the difference in the volume of this duct and that in Example 3 is not significant when compared to the total enclosure volume of 4.83 cu. ft., no changes need be made in the design. This system is to be used with an amplifier giving 6 dB boost at 38.58 Hz. Frequency response curves of the unassisted enclosure and loudspeaker are shown in Figure 16 along with the filter on its own and the combined system. Although ripple is present, this should be a good sounding system. The value of f_3 can be seen to be slightly lower than the approximation given in (44). The response of this particular system would be flatter and f_3 lower if the filter were to peak at a lower frequency, e.g. at f_B or lower. This is usually true for enclosures larger than required for a maximally flat sixth order alignment. The effect on this system of lowering f_P to 30 Hz is shown in Figure 17. This kind of manipulation is difficult to predict in a general manner and may lead to systems susceptible to excursion damage as in this case with $f_P < f_B$.

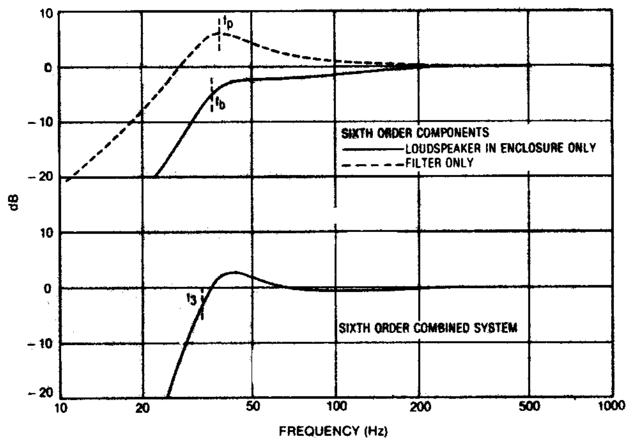

Figure 16. A Sixth Order System using the Same Loudspeaker as The Systems in Figure 15.

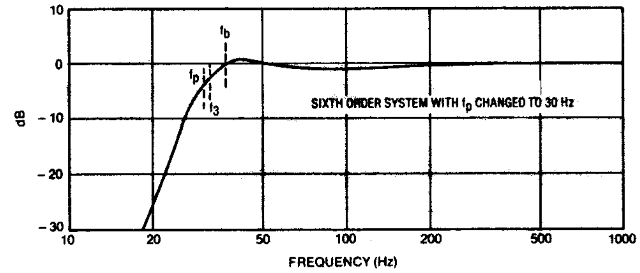

Figure 17. The Sixth Order System of Figure 15 with a Different (and Probably Dangerous) f_p.

References

[1] A. N. Thiele, "Loudspeakers in Vented Boxes", Audio Eng. Soc. 19, Part 1, pp. 382-391 (May 1971).

[2] D. B. Keele, "A New Set of Sixth-Order Vented-Box Loudspeaker System Alignments", J. Audio Eng. Soc. 23, 5, pp. 354-360 (June 1975).

Equations used to generate Figures 1, 2, 3, 4, 6, 8 and 9 are given below.

$$V_B/V_{AS} = a\,Q_T^{\,b}, \quad \text{(A1)}$$

$$f_B/f_S = c\,Q_T^{\,-d}, \quad \text{(A2)}$$

$$f_3/f_S = E\,Q_T^{\,-g}, \quad \text{(A3)}$$

where $a = 6.096,$ (A4)

$b = 2.432,$ (A5)

for $Q_T \leqslant 0.313.$ (A6)

$c = 0.3985,$ (A7)

$d = 0.940,$ (A8)

$E = 0.379,$ (A9)

$g = 1.143,$ (A10)

for $Q_T \leqslant 0.259.$ (A11)

$c = 0.428,$ (A12)

$d = 0.883.$ (A13)

$E = 0.218,$ (A14)

$g = 0.586,$ (A15)

for $0.259 < Q_T \leqslant 0.383.$ (A16)

$a = 18.029,$ (A17)

$b = 3.365,$ (A18)

for $0.313 < Q_T \leqslant 0.49.$ (A19)

$c = 0.397,$ (A20)

$d = 0.963,$ (A21)

$E = 0.233,$ (A22)

$g = 1.494,$ (A23)

for $0.383 < Q_T \leqslant 0.49.$ (A24)

$a = 6.427,$ (A25)

$b = 1.943,$ (A26)

$c = 0.457,$ (A27)

$d = 0.766,$ (A28)

$E = 0.353,$ (A29)

$g = 0.902,$ (A30)

for $Q_T > 0.49.$ (A31)

Equations for Figure 4 follow

$$A_P = (R - B)/2X^2 \text{ in.}^2, \quad \text{(A32)}$$

where $R = \sqrt{B^2 - 4X^2t^2}$ (A33)

$B = 2t + 0.9216$ (A34)

and $X = 4.5755 \times 10^6/(12^3 \times V_B \times f_B^{\,2}).$ (A35)

V_B and f_B are the volume in cubic ft. and the enclosure tuning frequency in Hz. Also, t is the baffle thickness in inches.

The equation for Figure 6 is

$$l = (A_D \times (t + 0.96\sqrt{A_P}))/A_P - 0.96\sqrt{A_P}, \quad \text{(A36)}$$

where l is the duct length in inches and A_P and A_D are the required duct area and the port area in square inches.

Equations to generate Figure 8 and to approximate 9 follow.

$$f_B/f_S = c(aV_S/V_B)d/b, \quad \text{(A37)}$$

$$f_3/f_S = E(aV_S/V_B)^{g/b} \quad \text{(A38)}$$

where a, b, c, d, E and g are given above. It can be seen from Figure 9 that equation (A38) fails for loudspeakers with Q_T less than 0.3 and enclosures larger than about 1.5 times the maximally flat volume.

Glossary of Important Thiele-Small Symbols

α — system compliance ratio, $= V_{AS}/V_B$

B — magnetic flux density in driver air gap

c — velocity of sound in air (= 345 m/s)

C_{AB} — acoustic compliance of air in enclosure

C_{AS} — acoustic compliance of driver suspension

C_{AT} — total acoustic compliance of driver and enclosure

C_{MEC} — electrical capacitance representing moving mass of system $(= M_{AC}S_D{}^2/B^2l^2)$

e_g — open-circuit output voltage of source (Thevenin's equivalent generator for amplifier output port)

f — natural frequency variable

f_B — resonance frequency of vented enclosure

f_C — resonance frequency of closed-box system

f_{CT} — resonance frequency of driver in closed, unfilled, unlined test enclosure

f_H — frequency of upper voice-coil impedance peak

f_L — frequency of lower voice-coil impedance peak

f_M — frequency of minimum voice-coil impedance between f_L and f_H ($f_M = f_B$)

f_S — resonance frequency of unenclosed driver

f_{SB} — effective value of f_S when mounted in an enclosure (is enclosure dependant)

f_3 — half-power (−3 dB) frequency of loudspeaker system response

G(s) — response function

h — system tuning ratio, $= f_B/f_S$

k — efficiency constant

k_p — power rating constant

k_x — displacement constant

l — length of voice-coil conductor in magnetic gap

L_{CET} — electrical inductance representing total system compliance $(= C_{AT}B^2l^2/S_D{}^2)$

M_{AC} — acoustic mass of driver in enclosure including air load

M_{AS} — acoustic mass of driver diaphragm assembly including air load

P_{AR} — displacement-limited acoustic power rating

P_{ER} — displacement-limited electrical power rating

$P_{E(max)}$ — thermally-limited maximum input power

Q — ratio of energy stored to energy dissipated per cycle ($Q = b/2a$ for $\ddot{x} + 2a\dot{x} + x\,b^2x = 0$)

Q_A — enclosure Q at f_B resulting from absorption losses

Q_B — total enclosure Q at f_B resulting from all enclosure and vent losses

Q_L — enclosure Q at f_B resulting from leakage losses

Q_P — enclosure Q at f_B resulting from vent frictional losses

Q_T — total driver Q at f_S resulting from all system resistances

Q_{EC} — Q of system at f_C considering electrical resistance R_E only

Q_{ES} — driver Q at f_S considering electrical resistance R_E only

Q_{MC} — Q of system at f_C considering system non-electrical resistances only

Q_{MS} — Q of system at f_S considering driver non-electrical resistances only

Q_{TC} — total Q of system at f_C including all system resistances

Q_{TCO} — value of Q_{TC} with $R_g = 0$

Glossary of Important Thiele-Small Symbols (continued)

Q_{TS} — total Q of driver at f_S considering all driver resistances

R_{AB} — acoustic resistance of enclosure losses caused by internal energy absorption

R_{AS} — acoustic resistance of driver suspension losses

R_E — dc resistance of driver voice coil

R_{ES} — electrical resistance representing driver suspension losses ($= B^2l^2/S_D{}^2R_{AS}$)

R_g — output resistance of source (Thevenin's equivalent resistance for amplifier output port)

s — complex frequency variable ($= \sigma + j\omega$)

S_D — effective surface area of driver diaphragm

T — radian time period ($= 1/2\ \pi\ f$) (time per radian)

U_o — system output volume velocity

V_{AB} — volume of air having same acoustic compliance as air in enclosure ($= \varrho_o c^2 C_{AB}$)

V_{AS} — volume of air having same acoustic compliance as driver suspension ($= \varrho_o c^2 C_{AS}$)

V_{AT} — total system compliance expressed as equivalent volume of air ($= \varrho_o c^2 C_{AT}$)

V_B — net internal volume of enclosure

V_D — peak displacement volume of driver diaphragm ($= S_D x_{max}$)

x_{max} — peak linear displacement of driver diaphragm

X(s) — displacement function

$Z_{VC}(s)$ — voice-coil impedance function

γ_B — ratio of specific heat at constant pressure to that at constant volume for air in enclosure

η_o — reference efficiency

ϱ_o — density of air ($= 1.18\ kg/m^3$)

ω — radian frequency variable ($= 2\ \pi\ f$)