ALTEC ENGINEERING NOTES

Technical Letter No. 258B

Uniformity of Coverage in Distributed Sound Systems

By
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Introduction

In the design of distributed sound systems, it is useful to know the uniformity of direct sound coverage at ear level for different combinations of loudspeaker type, pattern and degree of overlap. Conversely, it is also useful to know which pattern and overlap to use for a given speaker in order to achieve a predetermined or acceptable uniformity of coverage. As the choice of loudspeaker and pattern depend not only on uniformity of coverage but also on other design factors such as maximum SPL and cost, then it is also useful to know typical uniformities of coverage prior to loudspeaker selection. Uniformity of coverage can be related to total loudspeaker cost for a given speaker by showing the dependance of the maximum SPL variation on density in number of speakers per coverage circle area. Coverage circles for different Altec Lansing loudspeakers are discussed and values tabulated in Tech Letter 257.

Theory

For a regular array of n ceiling speakers shown diagramatically as a one dimensional model for clarity in Figure 1, the SPL at a general point P in the ear level plane is assumed to be caused by the addition of the intensities at P from all n loudspeakers, i.e.

$$SPL(P) = 10\,\text{Log}\,(\sum_{i=1}^{n} 10^{L_i/10}),\ i = 1,2,3,--- \quad (1)$$

L_i for a given speaker type can be calculated from the distance (r) between P and the projection of the centre of the i^{th} speaker onto the ear level plane from

$$L_i = -A\,[\arctan(r/h)]^B\ \text{dB} \quad (2)$$

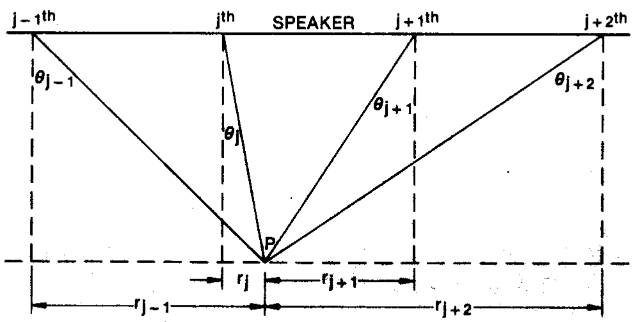

Figure 1. One dimensional representation of a regular ceiling array.

as explained in Tech Letter 257, where values of A and B are tabulated (as a and b) for a selection of Altec Lansing loudspeakers. Parameter h is the speaker height above ear level. To render such a system more tractable, it is further assumed that for all distant speakers for which r/h is greater than some limiting value (r'/h), the total intensity from these distant speakers only will show little variation with the position of P and hence can be regarded as a constant low level contribution to the direct sound.

For speakers at distances much greater than r(−6), the contributions are not only at a low level but are delayed and it can be argued that they more closely resemble early arrivals of the reverberant field. The value r(−6) is the radius on the ear level plane for which the SPL from a single speaker is 6 dB lower than the on-axis value. (Values of r(−6)/h for a selection of Altec Lansing loudspeakers are given in Tech Letter 257). The calculations were made on approximately square arrays of almost one thousand loudspeakers.

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Results

Values of the SPL at P were calculated from (1) and (2) for different positions of P within one unit cell of the loudspeaker pattern for five different loudspeakers in two frequency bands. This was carried out for the two traditional patterns (square and hexagonal) at three different degrees of overlap, namely, edge to edge, minimum and edge to centre (sometimes referred to as centre to centre). The sound levels for a typical speaker relative to the value on axis of a single loudspeaker are given for one unit cell in Figures 2 through 7. These values are the mean of the five speaker types and two different one-third octave bands (2 kHz and 4 kHz). For specific speakers in these frequency bands, individual unit cell sound patterns are given in Appendix I.

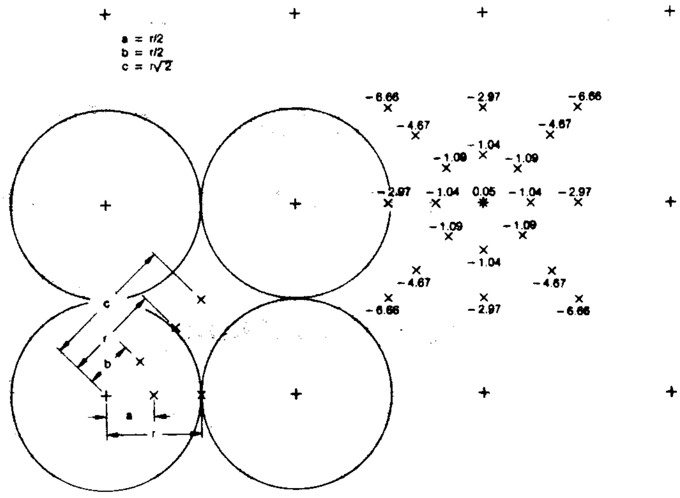

Figure 2. Typical relative SPL value for an edge to edge square configuration.

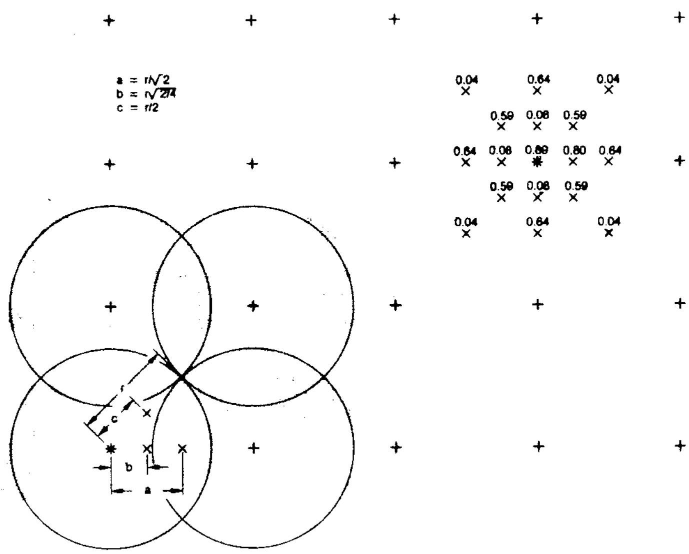

Figure 3. Typical relative SPL values for a minimum overlap square configuration.

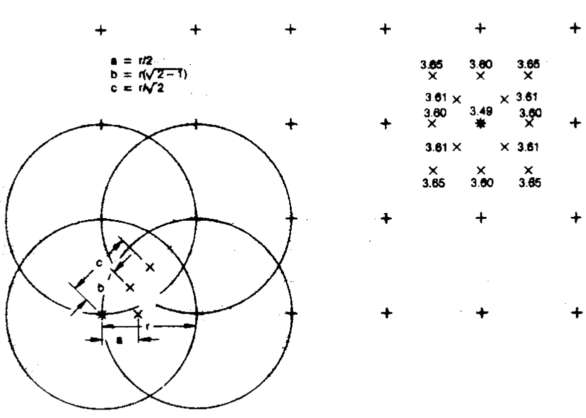

Figure 4. Typical relative SPL values for an edge to centre square configuration.

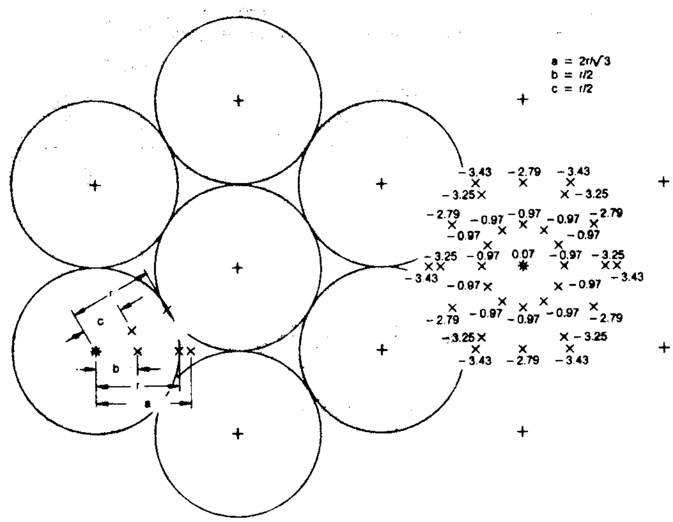

Figure 5. Typical relative SPL values for an edge to edge hexagonal configuration.

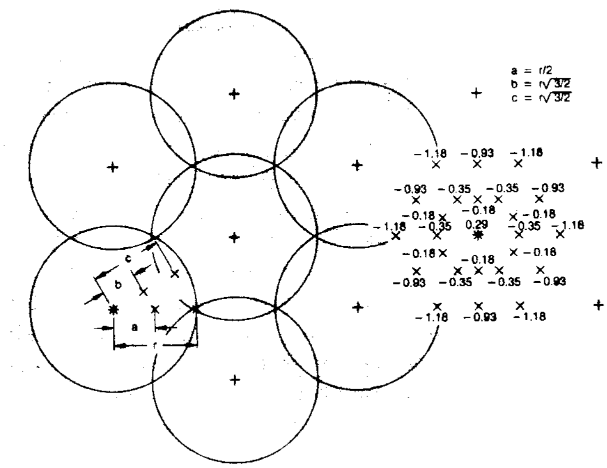

Figure 6. Typical relative SPL values for a minimum overlap hexagonal configuration.

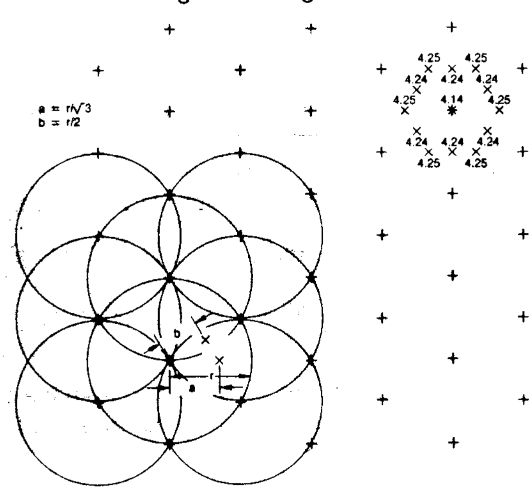

Figure 7. Typical relative SPL values for an edge to centre hexagonal configuration.

Values of the maximum SPL (L_{max}) and the minimum SPL (L_{min}) relative to the axial SPL of a single speaker for the typical loudspeaker are given in Table I for different combinations of pattern and overlap. Table I also lists the difference between L_{max} and L_{min} along with the areal density (σ) in number of speakers per −6 dB coverage circle area. Similar values for individual loudspeakers are given in Appendix II.

Table I. Values of speaker density (No. per coverage circle area), L_{max}, L_{min}, and $L_{max} - L_{min}$ for different combinations of pattern and overlap using a typical loudspeaker.

Pattern	Density	L_{max}(dB)	L_{min}(dB)	$L_{max} - L_{min}$(dB)
Edge to Edge Square	0.785	0.05	−6.66	6.71
Edge to Edge Hexagonal	0.907	0.07	−3.43	3.50
Min. Overlap Square	1.571	0.89	0.04	0.85
Min. Overlap Hexagonal	1.209	0.29	−1.18	1.47
Edge to Centre Square	3.142	3.65	3.49	0.16
Edge to Centre Hex.	3.628	4.25	4.14	0.11

The values in Table I can be approximated by an empirical equation of the form

$$L = a + b \operatorname{Ln} \sigma \text{ dB} \tag{3}$$

or alternatively

$$L = a + b' \operatorname{Log} \sigma \text{ dB} \tag{4}$$

where

$$b' = b \operatorname{Ln} 10 \tag{5a}$$

or

$$b' = b / \operatorname{Log} e. \tag{5b}$$

Figure 8 shows the dependence of L_{max} and L_{min} from Table I on density. Also shown are curves of the form given by (3) and (4). For L_{max},

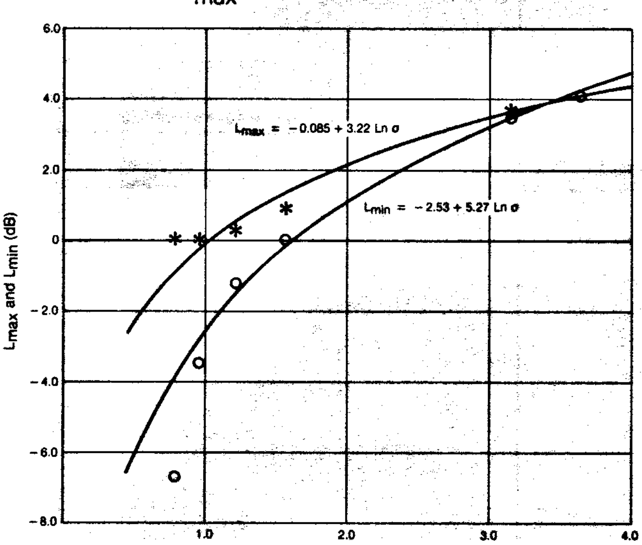

Figure 8. The dependence of L_{max} and L_{min} on density.

$$L_{max} = -0.085 + 3.22 \operatorname{Ln} \sigma \text{ dB} \tag{6a}$$

or

$$L_{max} = -0.085 + 7.41 \operatorname{Log} \sigma \text{ dB}. \tag{6b}$$

For L_{min}, the value for the edge to edge hexagonal pattern was omitted from the analysis. The equations found were

$$L_{min} = -2.53 + 5.27 \operatorname{Ln} \sigma \text{ dB} \tag{7a}$$

or

$$L_{min} = -2.53 + 12.13 \operatorname{Log} \sigma \text{ dB}. \tag{7b}$$

The dependence of the vales of $L_{max} - L_{min}$ from Table I on density is shown in Figure 9. The appropriate empirical equation also shown in Figure 9 was found to be

$$L_{max} - L_{min} = 2.44 - 2.05 \operatorname{Ln} \sigma \text{ dB} \tag{8a}$$

or

$$L_{max} - L_{min} = 2.44 - 5.76 \operatorname{Log} \sigma \text{ dB}. \tag{8b}$$

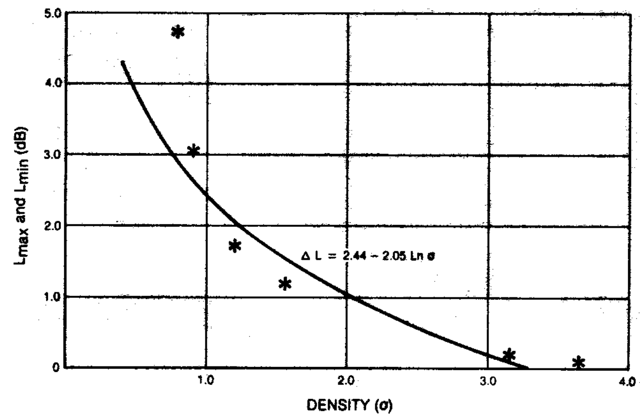

Figure 9. The dependence of $L_{max} - L_{min}$ on density.

The value of these equations is that they enable L_{max}, L_{min} and L_{max}-L_{min} to be determined for loudspeaker densities other than those associated with the traditional speaker pattern and overlap combinations. For example, configurations might be useful which lie in the present large gap between densities of $\pi/2$ and π. This gap is obvious from Figure 8. Methods of calculating the density are indicated in Appendix III.

So that relationships of the kind given in (3) and (4) can be used for specific loudspeaker types, values of a, b and b′ for calculating L_{max}, L_{min} and L_{max}-L_{min} are tabulated in Appendix IV for several Altec Lansing loudspeakers for the 2 kHz and 4 kHz one-third octave bands.

Equation (3) can be written in the form

$$\sigma = e^{(L-a)/b} \tag{9}$$

also

$$\sigma = \pi r^2(-6)/x^2 \tag{10}$$

where r(−6) is the radius of the planar −6dB coverage circle, hence

$$x = r(-6)\left(\pi e^{-(L-a)/b}\right)^{0.5} \tag{11}$$

which can be written as

$$x/h = c e^{-L/2b} \quad (12)$$

where $$c = [r(-6)/h](\pi e^{a/b})^{0.5} \quad (13)$$

Using values of a and b from Appendix IV and values of r(−6)/h from Tech Letter 257, values of c can be calculated and are also given in Appendix IV along with a, b and b′.

Appendix I

The next few pages give the sound distribution in one cell in the ear-level plane of distributed systems of loudspeakers arranged in the conventional patterns and degrees of overlap. The point locations are the same as those in the corresponding positions in Figures 2 through 7 except that all hexagonal patterns here are rotated by 90°.

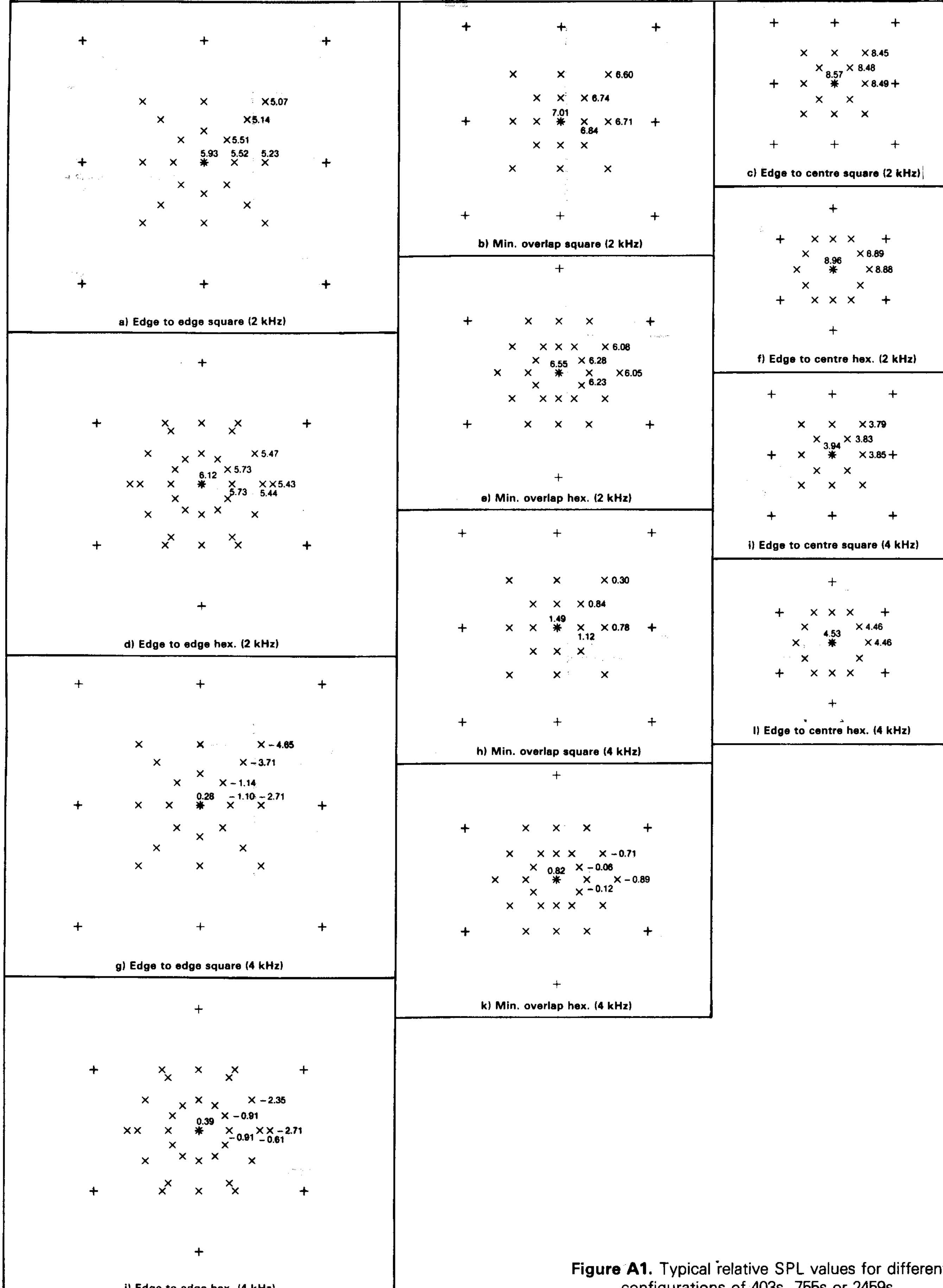

Figure A1. Typical relative SPL values for different configurations of 403s, 755s or 2459s.

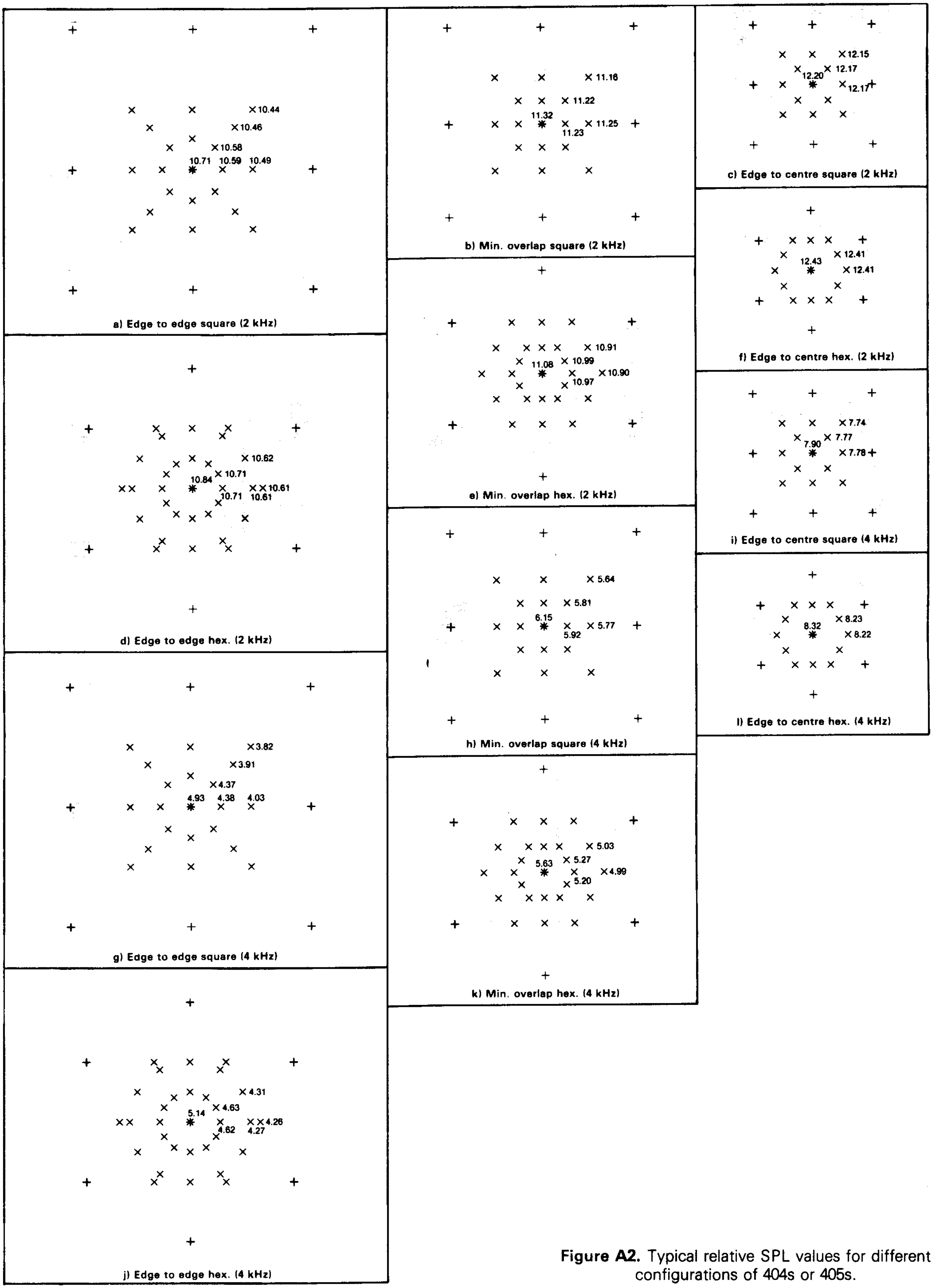

Figure A2. Typical relative SPL values for different configurations of 404s or 405s.

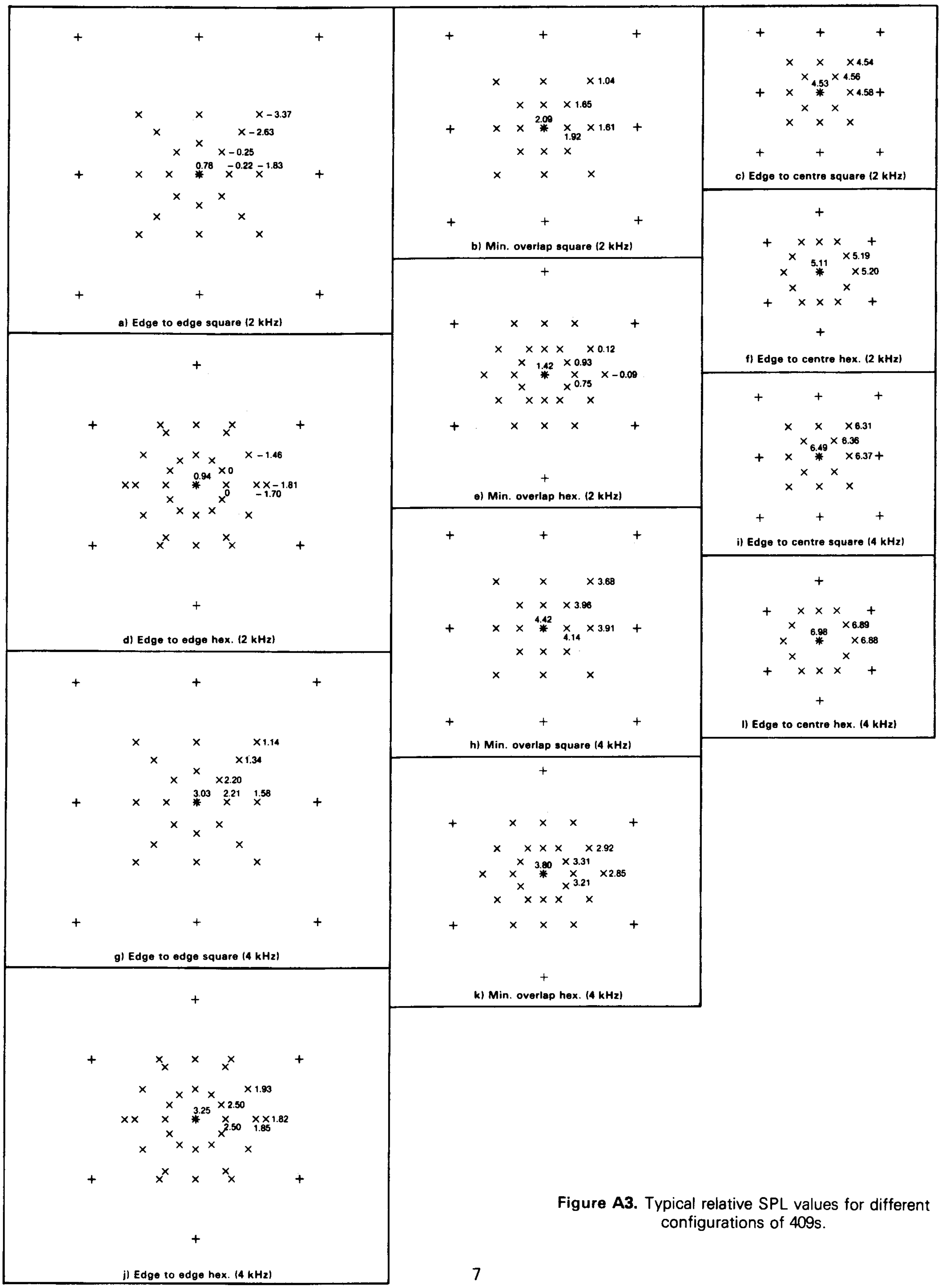

Figure A3. Typical relative SPL values for different configurations of 409s.

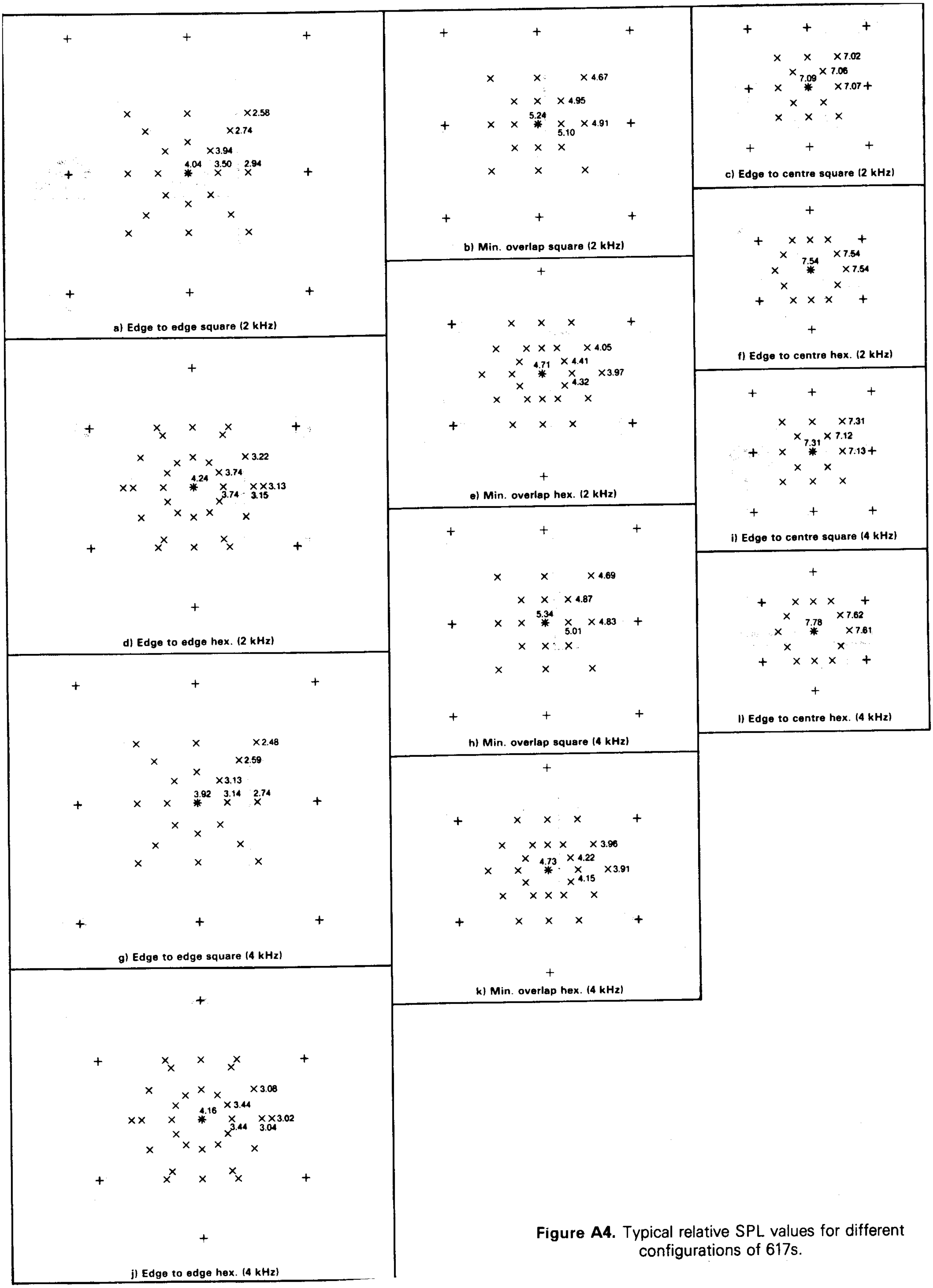

Figure A4. Typical relative SPL values for different configurations of 617s.

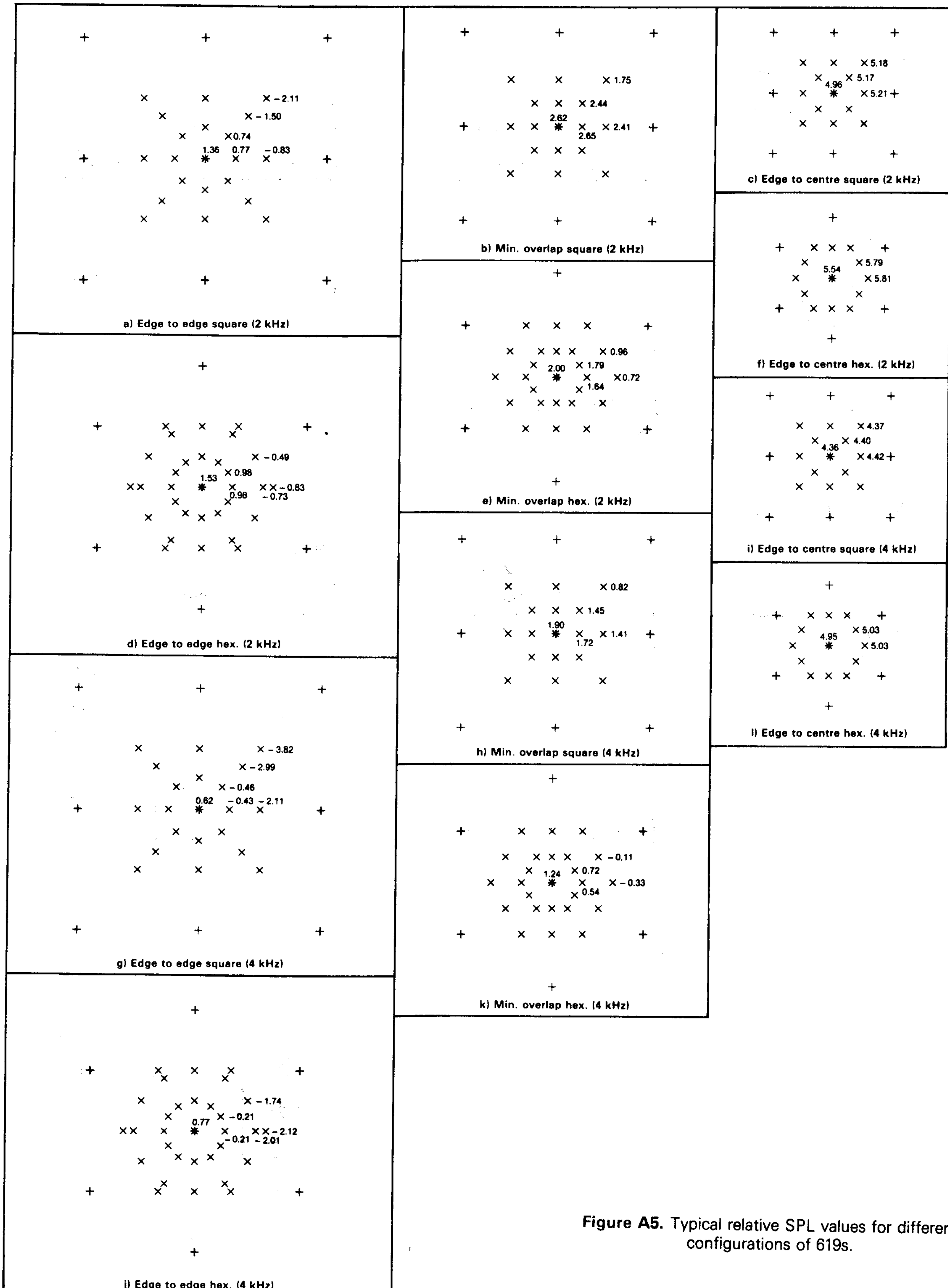

Figure A5. Typical relative SPL values for different configurations of 619s.

Table A1. L_{max} L_{min}, L_{max}-L_{min} for 403s, 755s and 2459s at 2 kHz and 4 kHz.

Pattern	2 kHz L_{max}	2 kHz L_{min}	2 kHz L_{max}-L_{min}	4 kHz L_{max}	4 kHz L_{min}	4 kHz L_{max}-L_{min}
Edge to Edge Square	5.93	5.07	0.86	0.28	−4.65	4.93
Edge to Edge Hexagonal	6.12	5.43	0.69	0.39	−2.71	3.10
Min. Overlap Square	7.01	6.60	0.41	1.49	0.30	1.19
Min. Overlap Hexagonal	6.55	6.05	0.50	0.82	−0.71	1.53
Edge to Centre Square	8.57	8.45	0.12	3.94	3.79	0.15
Edge to Centre Hex.	8.96	8.88	0.08	4.53	4.46	0.07

Table A2. L_{max} L_{min}, L_{max}-L_{min} for 404s and 405s.

Pattern	2 kHz L_{max}	2 kHz L_{min}	2 kHz L_{max}-L_{min}	4 kHz L_{max}	4 kHz L_{min}	4 kHz L_{max}-L_{min}
Edge to Edge Square	10.71	10.44	0.26	4.93	3.82	1.11
Edge to Edge Hexagonal	10.84	10.61	0.23	5.14	4.26	0.88
Min. Overlap Square	11.32	11.16	0.16	6.15	5.64	0.51
Min. Overlap Hexagonal	11.08	10.90	0.18	5.63	4.99	0.64
Edge to Centre Square	12.20	12.15	0.05	7.90	7.74	0.26
Edge to Centre Hex.	12.43	12.41	0.02	8.32	8.22	0.10

Table A3. L_{max} L_{min}, L_{max}-L_{min} for 409s.

Pattern	2 kHz L_{max}	2 kHz L_{min}	2 kHz L_{max}-L_{min}	4 kHz L_{max}	4 kHz L_{min}	4 kHz L_{max}-L_{min}
Edge to Edge Square	0.78	−3.37	4.15	3.03	1.14	1.89
Edge to Edge Hexagonal	0.94	−1.81	2.75	3.25	1.82	1.43
Min. Overlap Square	2.09	1.04	1.05	4.42	3.68	0.74
Min. Overlap Hexagonal	1.42	−0.09	1.51	3.80	2.85	0.95
Edge to Centre Square	4.58	4.53	0.05	6.49	6.31	0.18
Edge to Centre Hex.	5.20	5.11	0.09	6.98	6.88	0.10

Table A4. L_{max} L_{min}, L_{max}-L_{min} for 617s.

Pattern	Frequency 2 kHz L_{max}	L_{min}	L_{max}-L_{min}	4 kHz L_{max}	L_{min}	L_{max}-L_{min}
Edge to Edge Square	4.04	2.58	1.46	3.92	2.48	1.44
Edge to Edge Hexagonal	4.24	3.13	1.11	4.16	3.02	1.14
Min. Overlap Square	5.24	4.67	0.57	5.34	4.69	0.65
Min. Overlap Hexagonal	4.71	3.97	0.74	4.73	3.91	0.82
Edge to Centre Square	7.09	7.02	0.07	7.31	7.12	0.19
Edge to Centre Hex.	7.54	7.54	0.00	7.78	7.61	0.17

Table A5. L_{max} L_{min}, L_{max}-L_{min} for 619s.

Pattern	Frequency 2 kHz L_{max}	L_{min}	L_{max}-L_{min}	4 kHz L_{max}	L_{min}	L_{max}-L_{min}
Edge to Edge Square	1.36	−2.11	3.47	0.62	−3.82	4.44
Edge to Edge Hexagonal	1.53	−0.83	2.36	0.77	−2.12	2.89
Min. Overlap Square	2.65	1.75	0.90	1.90	0.82	1.08
Min. Overlap Hexagonal	2.00	0.72	1.28	1.24	−0.33	1.57
Edge to Centre Square	5.21	4.96	0.25	4.42	4.36	0.06
Edge to Centre Hex.	5.81	5.54	0.27	5.03	4.95	0.08

In order to calculate the mean speaker density of any regular array, a unit cell with a speaker axis at the centre must first be identified. This cell will always be a square or a regular hexagon (for the same spacing along rows in different directions). The cell sides are the perpendicular bisectors of line segments joining adjacent speaker positions. The area (S) of the cell can be found in terms of r^2. A diagonal or a perpendicular to one side can usually be related to r. This area, when divided by the area of the coverage circle yields the area (in coverage circle area units) per speaker. The required density is the reciprocal of this, i.e.

$$\sigma = \pi r^2/S(r^2). \qquad (A1)$$

Examples:

1) Edge to edge hexagon

From Figure A6 the unit cell is a regular hexagon with perpendicular of length r from the centre to each side. The unit cell area is given by

$$S = 2r^2\sqrt{3}. \qquad (A2)$$

Substituting into (A1) yields

$$\sigma = \pi\sqrt{3}/6. \qquad (A3)$$

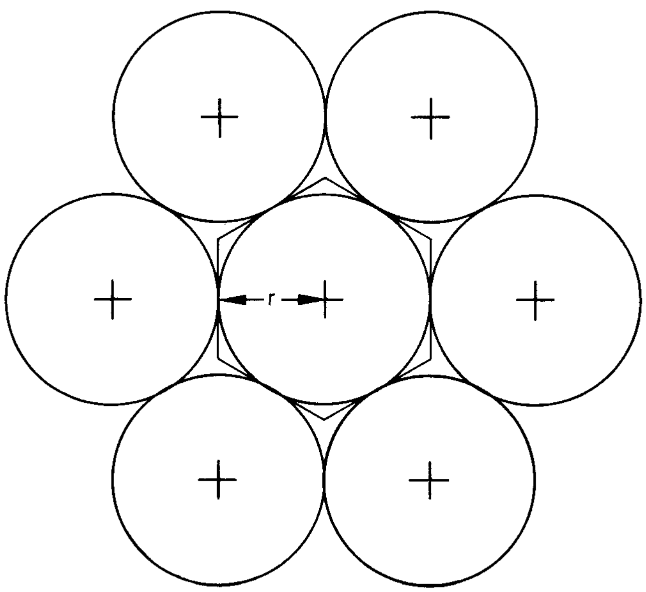

Figure A6. Unit cell for the edge to edge hexagonal pattern.

2) Edge to centre square

From Figure A7, the unit cell is a square of side length r.

$$S = r^2 \qquad (A4)$$

therefore $\sigma = \pi$. (A5)

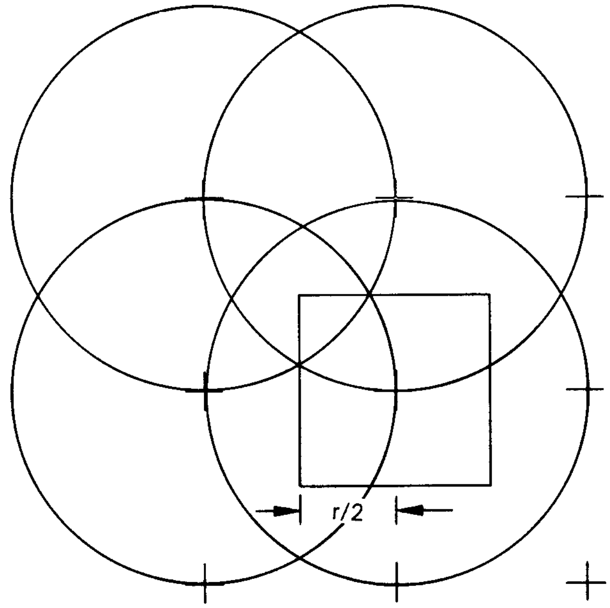

Figure A7. Unit cell for the edge to centre square pattern.

3) A hexagonal pattern with centres at one third of a diameter spacing.

From Figure A8, the unit cell of this non-standard configuration is a hexagon with a diagonal length of 2r/3.

$$S = 2r^2\sqrt{3}/3$$

therefore $\sigma = 3\pi r^2/2r^2\sqrt{3}$

$= 3\pi\sqrt{3}/2$

$= 8.162.$

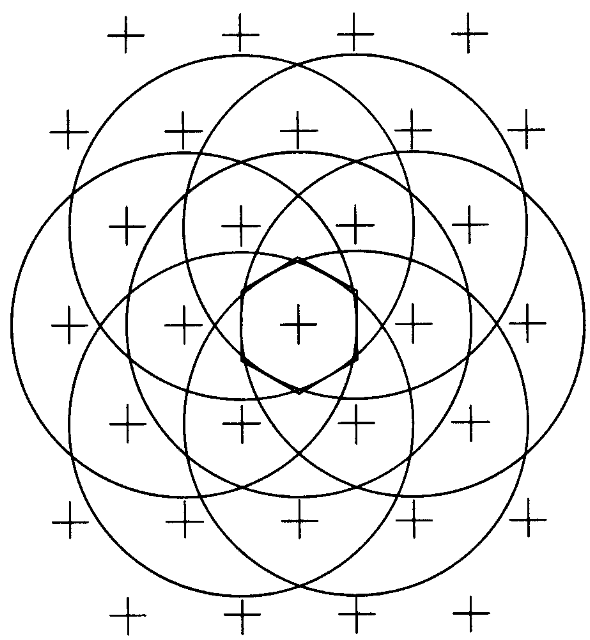

Figure A8. Unit cell for hexagonal pattern with one-third of a diameter spacing.

4) To find a square pattern with L_{min} approximately equal to 2.0 dB (extra gain needed for a hypothetical case). Let the spacing be x.

From (7a)

$$\sigma = e^{(2.0 + 2.53)/5.27} \quad \text{(A8)}$$

$$= 2.36 \quad \text{(A9)}$$

therefore $2.36 = \pi r^2/x^2$ (A10)

hence

$$x = r\sqrt{\pi/2.36}$$

$$= 1.15r. \quad \text{(A11)}$$

Also, from (6a),

$$L_{max} = -0.085 + 3.22 \, \text{Ln}\, 2.36 \text{ dB} \quad \text{(A12)}$$

$$= 2.68 \text{ dB}. \quad \text{(A13)}$$

A system could be similarly designed from a specified L_{max} using (6a) or (6b) or also for a specified L_{max}-L_{min} using (8a) or (8b) to determine the density.

Values of a, b and b′ are given here to enable L_{max}, L_{min} or L_{max}-L_{min} to be found from equations of the kind given in (3) or (4) and x/h to be found from (12) and 4 kHz one-third octave bands.

Table A6. Values of a, b,b′ and c for evaluating L_{max} and x/h from L_{max}.

	Frequency							
	2 kHz				4 kHz			
Speaker type	a	b	b′	c	a	b	b′	c
403, 755, 2459	6.28	2.00	4.61	6.82	0.61	2.69	6.19	0.69
404, 405	10.92	1.12	2.58	2.21	5.32	2.24	5.16	4.01
409	1.15	2.97	6.84	1.79	3.66	2.63	6.06	2.33
617	4.42	2.32	5.34	4.18	4.37	2.56	5.89	2.46
619	1.73	3.00	6.91	3.81	0.98	2.97	6.84	1.65

Table A7. Values of a, b, b′ and c for evaluating L_{min} and x/h from L_{min}.

	Frequency							
	2 kHz				4 kHz			
Speaker type	a	b	b′	c	a	b	b′	c
403, 755, 2459	5.62	2.48	5.71	4.40	−2.45	5.54	12.76	0.50
404, 405	10.70	1.28	2.95	110.0	4.48	2.85	6.56	2.68
409	−1.51	5.29	12.18	1.28	2.10	3.69	8.50	1.60
617	3.36	3.20	7.37	2.73	3.28	3.34	7.69	1.70
619	−0.52	4.80	11.05	2.70	−1.84	5.44	12.53	1.18

Table A8. Values of a, b, b′ and c for evaluating $L_{max} - L_{min}$ and x/h from $L_{max} - L_{min}$.

	Frequency							
	2 kHz				4 kHz			
Speaker type	a	b	b′	c	a	b	b′	c
403, 755, 2459	0.66	−0.48	−1.11	0.71	3.06	−2.85	−6.56	0.36
404, 405	0.22	−0.16	−0.37	0.85	0.84	−0.61	−1.40	0.61
409	2.66	−2.32	−5.34	0.83	1.36	−1.06	−2.44	0.63
617	1.06	−0.88	−2.03	0.88	1.09	−0.78	−1.80	0.52
619	2.25	−1.80	−4.14	1.53	2.82	−2.47	−5.69	0.79